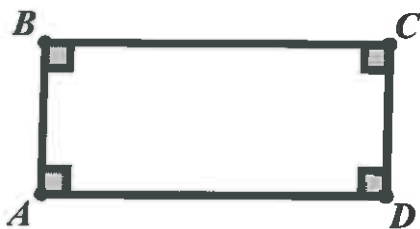


## Special Parallelograms: Rectangle – Rhombus – Square

**Recall:** State the 5 properties that can be used to identify a *parallelogram*:

1. Opp. sides are  $\parallel$ .
2. Opp. sides are  $\cong$ .
3. opp.  $\angle$ 's are  $\cong$ .
4. consecutive  $\angle$ 's are supp.
5. Diagonals bisect each other.

**Rectangle** - A quadrilateral with 4 congruent angles (or 4 right angles).



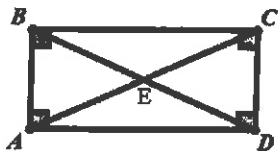
Pictured is rectangle ABCD. Based on the given definition of a rectangle, how do you know ABCD is also a *parallelogram*? Explain your reasoning.

The opp.  $\angle$ 's are  $\cong$ , thus ABCD must be a  $\parallel$ -gram.

( $\angle A \cong \angle C$  and  $\angle B \cong \angle D$ ).

### The Diagonals of a Rectangle:

1. The diagonals of a Rectangle bisect each other.



Given: Rectangle ABCD.

Explain why  $\overline{BD}$  and  $\overline{AC}$  bisect each other at E.

$\overline{BD}$  and  $\overline{AC}$  bisect each other because ABCD is also a  $\parallel$ -gram.

(the diagonals of a  $\parallel$ -gram bisect each other)

Now that you know

1.  $\overline{BD}$  and  $\overline{AC}$  bisect each other at E

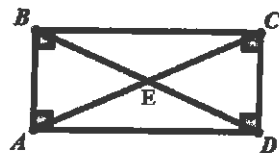
and

2.  $\overline{BD} \cong \overline{AC}$

what conclusion can you make about  $\overline{AE}$ ,  $\overline{BE}$ ,  $\overline{CE}$  and  $\overline{DE}$ ? Explain your reasoning.

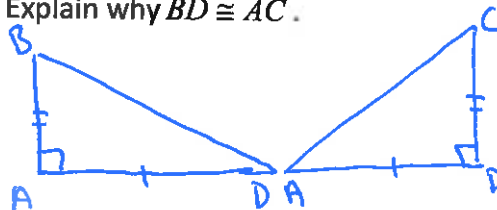
$\overline{AE} \cong \overline{BE} \cong \overline{CE} \cong \overline{DE}$  because halves of equals are equal.

2. The diagonals of a Rectangle are Congruent.



Given: Rectangle ABCD.

Explain why  $\overline{BD} \cong \overline{AC}$ .

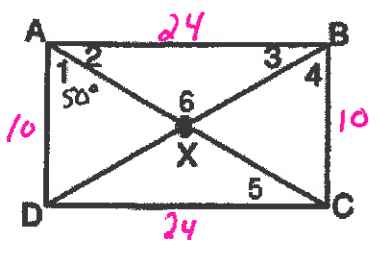


$\triangle ABD \cong \triangle DCA$  by SAS.

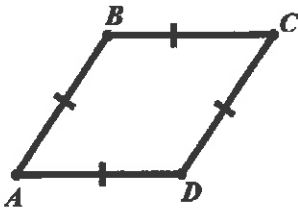
So,  $\overline{BD} \cong \overline{CA}$  by CPCTC.

## Using the Properties of a Rectangle:

Find each value and state the supporting property that justifies your conclusion.

	Find the values	State the reasons why?
<p>ABCD is a Rectangle.  <math>AB=24</math>, <math>BC=10</math>, <math>AC=26</math>,  <math>m\angle 1=50^\circ</math>.</p>	<p>a. <math>DC = 24</math>  <math>AD = 10</math></p>	<p>opp. sides of a rect are <math>\cong</math>.</p>
	<p>b. <math>DB = 26</math></p>	<p>Diags. of a Rect are <math>\cong</math>.</p>
	<p>c. <math>AX = 13</math>  <math>BX = 13</math></p>	<p>Diags of a Rect are <math>\cong</math> and bisect each other.</p>
	<p>d. <math>m\angle 2 = 40^\circ</math></p>	<p><math>\angle A = 90^\circ</math> and <math>\angle 1</math> and <math>\angle 2</math> are complementary.</p>
	<p>e. <math>m\angle 3 = 40^\circ</math></p>	<p>In <math>\triangle AXB</math>, since <math>AX = BX</math>, then the <math>\angle</math>'s across. are also <math>\cong</math>.</p>
	<p>f. <math>m\angle 6 = 100^\circ</math></p>	<p><math>\angle</math>'s of a <math>\triangle</math> sum to <math>180^\circ</math>.</p>
	<p>g. <math>m\angle 5 = 40^\circ</math></p>	<p><math>\angle 5 \cong \angle 2</math> because they are Alt. int.'s. and the opp sides of a rect are <math>\parallel</math>.</p>

**Rhombus** – A quadrilateral with 4 congruent sides.

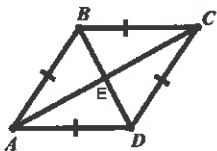


Pictured is **Rhombus** ABCD. Based on the given definition of a rhombus, how do you know ABCD is also a *parallelogram*? Explain your reasoning.

ABCD is also a  $\parallel$ -gram because its opposite sides are  $\cong$ .

## Diagonals of a Rhombus:

1. The diagonals of a Rhombus bisect each other.

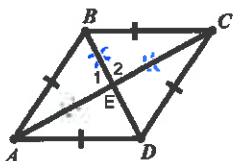


Given: Rhombus ABCD

Explain why  $\overline{BD}$  and  $\overline{AC}$  bisect each other at E.

Since ABCD is also a  $\parallel$ -gram, the diagonals bisect each other.

2. The diagonals of a Rhombus are perpendicular.



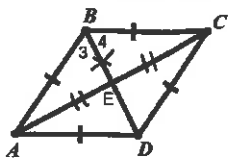
Given: Rhombus ABCD

Explain why  $\overline{BD} \perp \overline{AC}$ .

$\triangle ADE \cong \triangle CDE$  by SSS  
 $\angle 1 \cong \angle 2$  by CPCTC  
 $\angle$  supp  $\angle$  are  $\perp$ .

$2\angle$ 's both  $\cong$  + supp. are  $\perp$   $\angle$ 's so,  $\overline{AC} \perp \overline{BD}$ .

3. The diagonals of a Rhombus bisect its angles.



Given: Rhombus ABCD

Explain why  $\overline{BD}$  bisects  $\angle ABC$ .

$\triangle ABE \cong \triangle CBE$  by SSS

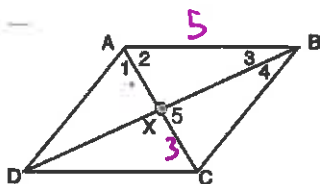
$\angle 3 \cong \angle 4$  by CPCTC

$\overline{BD}$  must bisect  $\angle ABC$ .

### Using the Properties of a Rhombus:

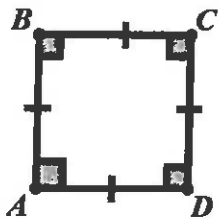
Find each value and state the supporting property that justifies your conclusion.

ABCD is a Rhombus.  
 $AB=5$ ,  $XC=3$ ,  $m\angle DAB=120^\circ$ .



Find the values	State the reasons why?
a. $DC = 5$ $AD = 5$	Rhombus has 4 $\cong$ sides.
b. $AX = 3$ $AC = 6$	Diagonals of Rhombus bisect each other.
c. $m\angle 2 = 60^\circ$	Diagonals of Rhombus bisect its $\angle$ 's.
d. $m\angle ABC = 60^\circ$	consec. $\angle$ 's of Rhombus are supp.
e. $m\angle 3 = 30^\circ$	Diags. of Rhombus bisect its $\angle$ 's.
f. $m\angle 5 = 90^\circ$	Diags of Rhombus are $\perp$ .
g. $BX = 4$	Pythagorean theorem. ( $\triangle AXB$ is Rt).

**Square** – A quadrilateral with 4 congruent sides and 4 congruent angles.



1. Is a Square also a Parallelogram? Explain your reasoning.

A square is also a parallelogram because its opp. sides are  $\cong$ .

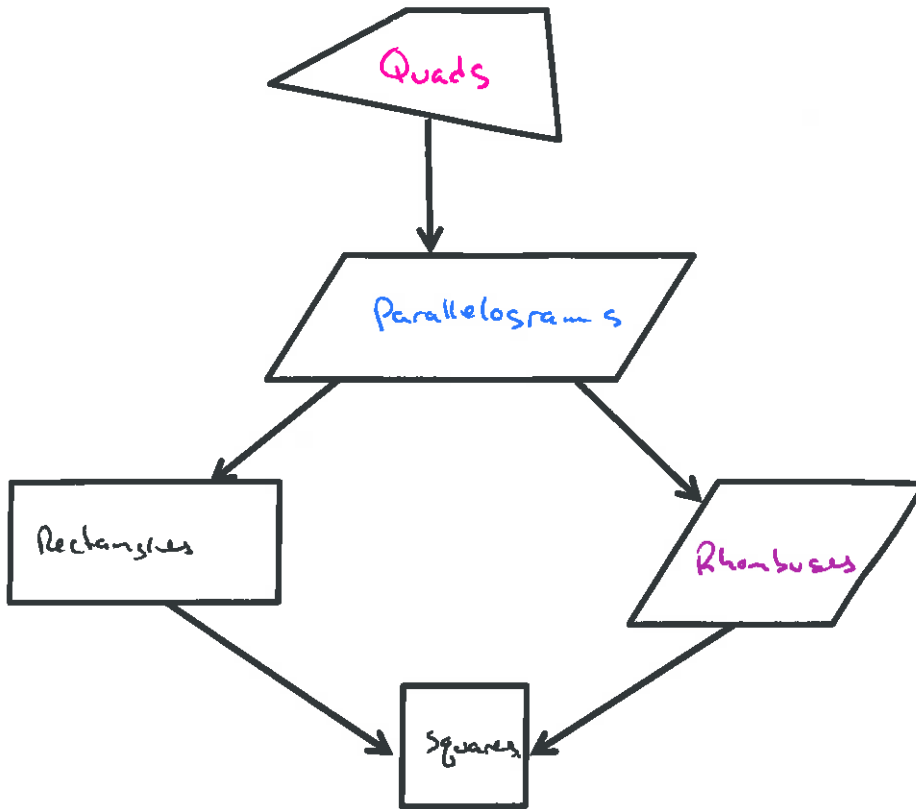
2. Is a Square also a Rectangle? Explain your reasoning.

A square is also a rectangle because it has 4  $\cong$   $\angle$ 's.

3. Is a Square also a Rhombus? Explain your reasoning.

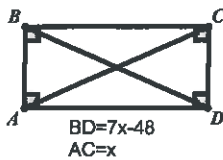
A square is also a rhombus because it has 4  $\cong$  sides.

# Quadrilateral Family Tree



**Examples:** Find the value of  $x$  in each special parallelogram. Provide a reason to support how you found  $x$ .

1.



Diags of a Rect are  $\cong$ .

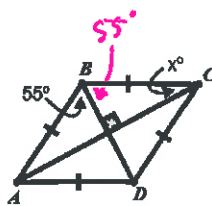
$$BD = AC$$

$$7x - 48 = x$$

$$6x = 48$$

$$x = 8$$

2.



Diags of Rhombus bisect its  $\angle$ 's.

Consecutive  $\angle$ 's of a Rhombus are supp.

$$m\angle ABC + m\angle BCD = 180$$

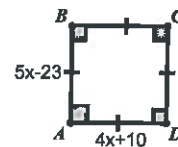
$$(55 + 55) + (x + x) = 180$$

$$110 + 2x = 180$$

$$2x = 70$$

$$x = 35$$

3.



Square has 4  $\cong$  sides.

$$AB = AD$$

$$5x - 23 = 4x + 10$$

$$x = 33$$